NAG Toolbox for MATLAB

f07gw

1 Purpose

f07gw computes the inverse of a complex Hermitian positive-definite matrix A, where A has been factorized by f07gr, using packed storage.

2 Syntax

```
[ap, info] = f07gw(uplo, n, ap)
```

3 Description

f07gw is used to compute the inverse of a complex Hermitian positive-definite matrix A, the function must be preceded by a call to f07gr, which computes the Cholesky factorization of A, using packed storage.

If **uplo** = 'U', $A = U^{\mathrm{H}}U$ and A^{-1} is computed by first inverting U and then forming $\left(U^{-1}\right)U^{-\mathrm{H}}$.

If **uplo** = 'L', $A = LL^{\mathrm{H}}$ and A^{-1} is computed by first inverting L and then forming $L^{-\mathrm{H}}\left(L^{-1}\right)$.

4 References

Du Croz J J and Higham N J 1992 Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates how A has been factorized.

uplo = 'U'

$A = U^{\mathrm{H}}U$, where U is upper triangular.

uplo = 'L'

$A = LL^{\mathrm{H}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **n – int32 scalar**

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

3: **ap**$(*)$ **– complex array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The Cholesky factor of A stored in packed form, as returned by f07gr.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **ap**$(*)$ **– complex array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The factorization contains the n by n matrix A^{-1}.

More precisely,

if **uplo** = 'U', the upper triangle of {\it A}$^{-1}$ must be stored with element A_{ij} in **ap**$(i+j(j-1)/2)$ for $i \leq j$;
if **uplo** = 'L', the lower triangle of {\it A}$^{-1}$ must be stored with element A_{ij} in **ap**$(i+(2n-j)(j-1)/2)$ for $i \geq j$.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows: 1: **uplo**, 2: **n**, 3: **ap**, 4: **info**.

info > 0

If **info** $= i$, the ith diagonal element of the Cholesky factor is zero; the Cholesky factor is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies

$$\|XA - I\|_2 \leq c(n)\epsilon\kappa_2(A) \qquad \text{and} \qquad \|AX - I\|_2 \leq c(n)\epsilon\kappa_2(A),$$

where $c(n)$ is a modest function of n, ϵ is the ***machine precision*** and $\kappa_2(A)$ is the condition number of A defined by

$$\kappa_2(A) = \|A\|_2 \left\|A^{-1}\right\|_2.$$

8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}n^3$.

The real analogue of this function is f07gj.

9 Example

```
uplo = 'L';
n = int32(4);
ap = [complex(1.797220075561143, +0);
     complex(0.8401864749527325, +1.068316577423342);
     complex(1.057188279741849, -0.467388502622712);
```

```
     complex(0.233694251311356, -1.391037210186643);
     complex(1.316353439509685, +0);
     complex(-0.4701749470106329, +0.3130658155999466);
     complex(0.08335250923944192, +0.03676071443037458);
     complex(1.560392977137124, +0);
     complex(0.9359617337923402, +0.9899692192815736);
     complex(0.6603332973655893, +0)];
[apOut, info] = f07gw(uplo, n, ap)
```

```
apOut =
   5.4691
  -1.2624 - 1.5491i
  -2.9746 - 0.9616i
   1.1962 + 2.9772i
   1.1024
   0.8989 - 0.5672i
  -0.9826 - 0.2566i
   2.1589
  -1.3756 - 1.4550i
   2.2934
info =
           0
```